Ioffe Institute
6th Int. Symp. "Nanostructures: Physics and Technology"
St Petersburg, Russia, June 22–26, 1998.

Electron transport in a lattice of dopped many-electrons quantum dots: a gapless Hubbard insulator

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Abstract. Electron states and hopping conductivity of a periodic lateral lattice of large quantum dots are studied taking into account the Coulomb effects. The intradot electron-electron repulsion produces the Hubbard gap which exceeds the single-electron levels spacing. The fluctuations of the number of impurities per a dot causes the redistribution of electrons and softens the Hubbard gap. The energy of interdot exitation varies from zero to the dot charging energy U_C. The variable range hopping with the typical hopping energy determined by U_C was demonstrated to be a predominant mechanism of low temperature transport.

Introduction

A periodic quantum dot lattice has brought a new goal for the solid state study, provided the artificial solid with controlled parameters such as the number of electrons per a cell, strength of electron-electron interaction, strength of interatomic tunnelling etc. Moreover, the scale of typical parameters has changed. The importance of electron-electron interaction gives rise to the Coulomb blockade, which is rather classical than quantum phenomenon.

This work was stimulated by some new experimental results on metal-non-metal transition at the crossover from antidots to quantum dots [1, 2]. The purpose of the present work is the theoretical study of electron states and hopping electron transport in a lattice of tunnel-coupled large quantum dots. We shall demonstrate that the intradot Coulomb interation leads to the appearance of Hubbard gap in the electron spectrum of the system. The fluctuations of the number of electrons per a dot smooths the gap up, producing the gapless Hubbard insulator. It will be shown that the activation energy of hopping conductivity is determined by the dot charging energy. The absence of hard gap tends to the variable range hopping in the low temperature case.

The typical sizes of modern dots lays between 100 and 500 nm. For usual electron density of 10^{11-12} cm^{-2} this gives 50–200 electrons per a dot. The other important parameter is the Coulomb energy per electron. It has the value of capacitive charging of a dot, $U_C = e^2/2C$, where C is the effective capacity of the order of the dot size. The typical value of U_C has the order of 1 meV. The large number of electrons determines a gaseous picture of electron distribution inside a dot. The typical Fermi energy referred to the dot bottom is 10 meV. The distance between energy levels is $\delta = E_F/n \sim 10^{-1}$ meV, where n is the mean number of electrons per a dot. The 2D screening length has the order of Bohr radius, $\sim$ 8.3 nm, which is sufficiently less than the dot size. Hence the dot has a plain bottom, strongly differs from the lattice of dots with parabolic confinement potential, considered by [3]. The other parameter, tunnelling factor t, is very easily controlled, varying from the maximal value $E_F/\sqrt{n}$ for barrierless dots to the exponentially small value.

Hence we shall deal with such hierarchy of parameters as

$$n \gg 1, \qquad E_F \gg U_C \gg \delta, \qquad t \ll U_C. \tag{1}$$

This combination of parameters is unusual. The large interaction makes inapplicable the single-electron models, like Anderson-Mott one, while the large number of electrons per a dot makes inapplicable the usual "single electron per single site" Hubbard model. The small ratio of interaction energy to the Fermi energy means near ideality of electron gas inside a dot, while large U_C/δ parameter determines the strong involvement of interaction into the energy levels distribution. The presence of tunnelling permits electron to propagate along the lattice of dots, while the small value of t determines that in the first approximation it does not affect on the electron states of a single dot.

Another complications is the large number of electron levels in a dot. It leads to the statistical picture of levels distribution, like Wigner-Dyson one. From the other hand, the levels of different dots are independently distributed, so this distribution does not affect to the electron jumps between dots.

The situation, when the Coulomb interaction in a dot is determinative for electron states and transport is typical for Coulomb blockade. The last deals with the tunnelling between Fermi lakes via single or few dots. In this case the finite Hubbard gap for transport appears. The difference of our problem from the mentioned one is caused by multiplication of quantum dots into the lattice.

Electron states

Let us consider electron states in a dot, neglecting tunnelling between dots. In the gateless system the number of electrons in a dot is determined by the number of the impurities in it. If this number is the same for all dots, say N, the system should be the Hubbard insulator with N-th filled Hubbard band. Really, N fluctuates. The fluctuation of this quantity has the order of statistical fluctuations $\sqrt{N}$. In the first approximation the large dot is electrically neutral and the number of electrons coincides with the number of impurities.

In the second approximation, we should take into account the fluctuations of the Fermi energy caused by the fluctuations of electron density. The value of these fluctuations is $\Delta = E_F/\sqrt{N}$. In equilibrium, the fluctuations of Fermi level are smoothed by the redistribution of electrons among dots and corresponding shifts of dot bottoms. This mechanism works if the number of electrons is large enough and Δ exceeds U_C. If Δ is less than U_C, the redistribution has no profit, giving loss in energy. If $\Delta \gg U_C$ (the case is typical), redistribution occurs. The number of redistributed electrons δn_j in a dot j is determined by the potential, necessary to equalize the local fluctuations of Fermi energy $\Delta_j = E_F(N_j - N)/N$. As a rule, the equation $e^2\delta n_j/2C = \Delta_j$ gives the fractional δn_j.

The discreeteness of δn_j tends to impossibility to equalize the remainder of order of potential, produced by a single electron per a dot. The residual local distances between the filled states and the Fermi level U_j are uniformly distributed within the range $(0, U_C)$.

In other words, the last occupied state can not be below the Fermi level more than $e^2/2C$, else the next level will be filled. The same is true for the first empty level: if it exceeds the Fermi energy more than $e^2/2C$, there should be another empty one, below the first on the distance $e^2/2C$. The first empty level in each dot is separated from the last filled by the distance U_C.

The activation energy for a jump between the fixed dots varies from 0 to $2U_C$. Hence the global energy gap vanishes, while local one is finite. We call this system *a gapless Hubbard insulator*.

The single-electron density of states for additional electron (hole) neglecting tunnelling linearly vanishes with the distance to the Fermi energy:

$$\rho(E) = \frac{S}{d^2}\frac{m}{\pi\hbar^2 e^2}2C|E-\mu|, \quad \text{if} \quad |E-\mu| < e^2/2C \tag{2}$$

and

$$\rho(E) = \frac{S}{d^2}\frac{m}{\pi\hbar^2}, \quad \text{if} \quad |E-\mu| > e^2/2C \tag{3}$$

where S is the quantum dot arrea, d is the lattice period. This behaviour, originating from *intrasite* e-e interaction, is similar to 2D Coulomb gap [4], caused by *intersite* interaction. Notice that their meanings are different.

The quantum dot lattice is an art-made system of Fermi lakes. Previously disordered system of Fermi lakes was discussed in the theory of strongly dopped and strongly compensated semiconductors [4]. In this situation the Hubbard gap is not much essential because it has the same order as the mean level spacing.

Electron transport

Below we shall neglect t in consideration of energy spectrum and take it into account to consider transport in the lattice. The conductivity of the system is determined by electron hoppings between dots. The problem may be separated on two parts, finding a tunnel exponent and hopping optimization. We studied the probability of tunneling by means of single electron Hamiltonian of Anderson model

$$H = \sum_{i,\alpha}\epsilon_i \hat{a}_i^\dagger \hat{a}_i + \sum_{i,j}\{t\hat{a}_i^\dagger \hat{a}_{j,\beta} + H.c.\} \tag{4}$$

Here $\epsilon_i = -U_i$ for filled $\epsilon_i = U_C - U_i$ for empty states. At high temperature the transitions between the nearest neighbors take place. At low temperatures the long range hopping is preferable due to minimization of activation energy. These hoppings can be performed by means of virtual transitions through the intermediate dots, accompanied by the emission (absorption) of phonons at the start or in the finish.

Optimization of hopping probability together with the construction of percolation network of limited strong hopping gives the conductivity:

$$\sigma \sim \exp\left[-(T_0/T)^{1/3}\right], \tag{5}$$

where $T_0 = 4\beta U_C \log^2(U_C/t)$, $\beta \sim 13.8$. The hopping length is

$$l_0 = d\left[\left(\frac{U_C}{T\log\frac{U_C}{t}}\right)^{1/3}\right]. \tag{6}$$

The square brackets denote the integer part.

The formula (6) is valid for low temperature region $T \ll U_C/(2\log U_C/t)$. For high temperatures an electron prefers to jump onto the nearest neighbore.

Hence the effective conductivity is

$$\sigma \sim (t/2U_C)^2 \quad \exp\left(-\frac{\sqrt{2}U_C}{3T}\right). \tag{7}$$

Discussion

We would like to compare the difference of transport in a quantum dot lattice and in impurity band. An impurity usually can confine a limited number of electrons. The Hubbard gap is usually too weak or too strong, so that the energy of A^+, A^0 and A^- states have very different scales. In the case of impurity system in semiconductors the upper Hubbard A^- state is known as an origin of so called "ϵ_2" conductivity. The energy of this state is very low, compared to the ground state of hydrogen-like impurities. In non-compensated donor system there are no long range Coulomb forces and the states of additional electrons (A^--states) and holes (A^+-states) on impurities are separated by a hard gap.

The principle feature of the quantum dot considered is a large number of electrons in a dot. As a result, the Hubbard energy is small, compared to not ionization bariers only, but to intradot Fermi energy. The electron gas *inside* quantum dot has a weak, perturbative interaction. But the Coulomb energy is rather strong if one consider transport between dots, for $U_C \gg t$.

An ideal quantum dot system should exhibit many Hubbard bands. The slow change of chemical potential leads to subsequent filling of subsequent Hubbard bands. So properties of the system should periodically alternate from metal to insulator. We expect this picture at least if tunnelling is strong enough and is not affected by phase-destroying thermal fluctuations.

The fluctuations of impurity numbers are much more important. If the fluctuations of local Fermi energy, caused by them, exceed the tunnelling amplitude, they result in the appearance of a gapless Hubbard insulator. When considering the gapless Hubbard insulator we neglect the influence of tunnelling on electron states, focusing on the case of hopping conductivity. We belive that the tunnelling may substantially change the properties of a gapless Hubbard insulator converting it to a "bad" metal.

The system considered is not exotic and is characterized by parameters, which are usual for lateral superlattices based on semiconductor heterostructures. It means the possibility of experimental realization of the predicted gapless Hubbard insulator.

Acknowledgments

We thank B. L. Altshuler for usefull discussions. The work was partially supported by Russian Foundation for Basic Researches (97-02-18397) and Folkswagen-Stiftung (Gr.I/71162). The full text of report is submitted to Europhys. Lett.

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